CHAPTER 3

Methods of Analysis

Here we apply the fundamental laws of circuit theory (Ohm's Law & Kirchhoff's Laws) to develop two powerful techniques for circuit analysis.
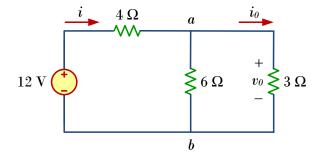1. Nodal Analysis (based on KCL)
2. Mesh Analysis (based on KVL)
This is the *most important* chapter for our course.

3.1. Nodal Analysis

Here, we analyze circuit using **node voltages** as the circuit variables.
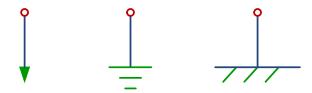
EXAMPLE 3.1.1. Back to Example 2.5.8

3.1.2. **Steps to Determine Node Voltages:**

Step 0: Determine the number of nodes n.

Step 1: Select a node as a reference node (ground node). Assign voltages v_1, v_2, $\cdots$, v_{n-1} to the remaining $n-1$ nodes.

- The voltage are now referenced with respect to the reference node.
- The ground node is assumed to have 0 potential.

- Recall that voltages are measured between two points. For node voltages, the second point is always the ground node.

Step 1s: If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node "equal" to the voltage of the voltage source.

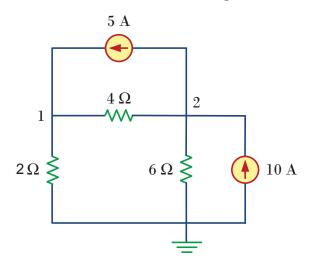**Step 2:** Apply KCL to each of the remaining nonreference nodes.

(a) Use Ohm's law to express the branch currents in terms of the node voltages.

(b) Current source automatically gives current value.

Caution: Watch out for the direction of the arrow.

Step 3: Solve the resulting *simultaneous equations* to obtain the unknown node voltages.
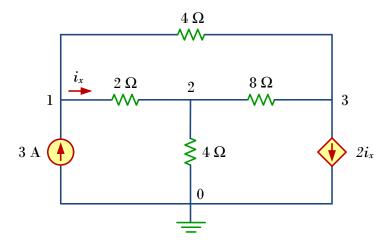
Step 4: After all the node voltages are determined, it is easy to find everything else.

3.1.3. Remarks: There are multiple methods to solve the simultaneous equations in Step 3.

- Method 1: Elimination technique (good for a few variables)
- Method 2: Write in term of matrix and vectors (write $\mathbf{Ax} = \mathbf{b}$), then use Cramer's rule.
- Method 3: Use
  - computer (e.g., MATLAB) to find A^{-1} and then find $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$
  - calculator (fx-991MS can solve simultaneous linear equations with two or three unknowns.)

EXAMPLE 3.1.4. Calculate the node voltages in the circuit below.
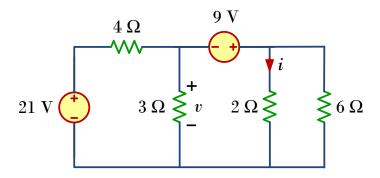

EXAMPLE 3.1.5. Calculate the node voltages in the circuit below.

The next example shows that the steps given in 3.1.2 are not sufficient to solve all interesting circuit-analysis problem.

EXAMPLE 3.1.6. Find v and i in the circuit below.

3.1.7. An extra step should be added to 3.1.2:

Step 1sn: If there is a voltage source connected between two nonreference nodes, the two nonreference nodes form a **supernode**. We apply both KCL and KVL to determine the node voltages.

3.1.8. Note the following properties of a supernode:

(a) The voltage source inside the supernode provides a constraint equation needed to solve for the node voltages.
(b) A supernode has no voltage of its own.
(c) We can have more than two nodes forming a single supernode.
(d) The supernodes are treated differently because nodal analysis requires knowing the current through each element. However, there is no way of knowing the current through a voltage source in advance.

3.2. Mesh Analysis

Mesh analysis provides another general procedure for analyzing circuits, using **mesh currents** as the circuit variables.

DEFINITION 3.2.1. **Mesh** is a loop which does not contain any other loop within it.

3.2.2. **Steps to Determine Mesh Currents:**
Step 0: Determine the number of meshes n.
Step 1: Assign mesh currents[1] i_1, i_2, ..., i_n, to the n meshes.
- The direction of the mesh current is arbitrary–(clockwise or counterclockwise)–and does not affect the validity of the solution.
- For convenience, we define currents flow in the clockwise (CW) direction.

Step 2a: From the current direction in each mesh, define the voltage drop polarities.
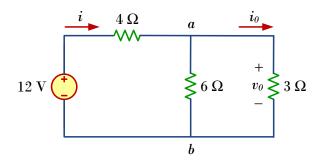**Step 2b:** Apply KVL to each of the n meshes.
   Use Ohm's law to express the voltages in terms of the mesh current.
- Tip (for combining Step 2a and 2b): Go around the loop in the same direction[2] as the mesh current (of that mesh). When we pass a resistor R, the voltage drops by $I \times R$ where I is the branch current (algebraic sum of mesh currents) through that resistor in the mesh-current direction.

Step 3: Solve the resulting n simultaneous equations for the mesh currents.
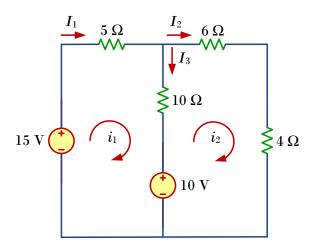**Step 4:** Other quantities related to the circuit can be found from the mesh currents.

---

[1]Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously.

[2]Note that according to our agreement above, this is in the CW direction.

EXAMPLE 3.2.3. Back to Example 2.5.8

EXAMPLE 3.2.4. Find the branch currents I_1, I_2, and I_3 using mesh analysis.

3.3. Remarks on Nodal Analysis and Mesh Analysis

3.3.1. Nodal analysis applies KCL to find unknown (node) voltages in a given circuit, while mesh analysis applies KVL to find unknown (mesh) currents.

3.3.2. Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is *planar*.

- A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is nonplanar.

3.3.3. **Nodal Analysis vs. Mesh Analysis**: Given a network to be analyzed, how do we know which method is better or more efficient?

Suggestion: You should be familiar with both methods. Choose the method that results in smaller number of variables or equations.

- A circuit with fewer nodes than meshes is better analyzed using nodal analysis, while a circuit with fewer meshes than nodes is better analyzed using mesh analysis.

You can also use one method to check your results of the other method.

3.3.4. Nodal analysis and mesh analysis can also be used to find equivalent resistance of a part of a circuit.

This becomes extremely useful when the techniques that we studied in the previous chapter cannot be directly applied (, e.g., when we can't find resistors that are in parallel or in series; they are all connected in some "strange" configuration.)

The are two approaches to this kind of problems.

(a) Apply 1 V voltage source across the terminals, find the corresponding current I through the voltage source. Then,

$$R_{eq} = \frac{V}{I} = \frac{1}{I}.$$

(b) Put 1 A current source through the terminals, find the corresponding voltage V across the current source. Then,

$$R_{eq} = \frac{V}{I} = \frac{V}{1} = V.$$

EXAMPLE 3.3.5. Find the equivalent resistance for the following circuit